

# Revision of exotic $0^{--}$ glueball

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We present the new results for the exotic glueball state  $0^{--}$  in the framework of the QCD sum rules. It is shown that previously used three-gluon current does not couple to any glueball bound state. We suggest considering a new current which couples to this exotic state. The resulting values for mass and decay constant of the  $0^{--}$  glueball state are  $M_G = 6.3_{-1.1}^{+0.8}$  GeV and  $F_G = 67 \pm 6$  keV, respectively.

The glueballs are composite particles that contain gluons and no valence quarks. The glueballs carry very important information about the gluonic sector of QCD and their study is one of the fundamental tasks for the strong interaction. While the glueballs are expected to exist in QCD theoretically, there was no clear experimental evidence and so the glueballs remain yet undiscovered (see reviews [1, 2]). This is the reason why the investigation of the possible glueball's candidates are included in the programs of the running and projected experiments such as Belle (Japan), BaBar (SLAC, USA), BESIII (Beijing, China), RHIC (Brookhaven, USA), LHC (CERN), GlueX (JLAB, USA), NICA (Dubna, Russia), HIAF (China) and FAIR (GSI, Germany).

One of the main problems of the glueball spectroscopy is the possible large mixing of the glueballs with ordinary meson states, which leads to the difficulties in disentangling the glueballs in the experiment. In this connection, the discovery of the exotic  $0^{--}$  glueball, which can not be mixed with the  $q\bar{q}$  states, is one of the fundamental tasks of the glueball spectroscopy. Therefore, it is very important to investigate the properties of this glueball within the QCD's based approach. One of such approaches is the QCD Sum Rules (SR). The first study of the  $0^{--}$  glueball by the QCD SR method has been performed recently in [3] where the authors introduced a very specific interpolating current for this three-gluon state. Unfortunately, they only considered SR for the mass of the glueball and did not check the SR for the decay constant. Below we show that their current has pathology, which leads to the negative sign of the imaginary part of the corresponding correlator and, as the result, SR become inconsistent. Considering the fact the study of the glueball is a very hot topic nowadays and the prediction of the value of the exotic glueball mass is crucial for the experimental observation, the revision of the exotic glueball properties within QCD SR is required.

In this Letter, a new interpolating current, which cou-

ples to the  $0^{--}$  exotic glueball state, is suggested. We calculate the Operator Product Expansion (OPE) for the correlator with this current up to dimension-8 and show that there is a good stability of SR for both mass and decay constant of this state.

The theoretical part of the QCD SR approach [4] to a bound state is the calculation of the OPE for the correlator defined by

$$\Pi(Q^2) = i \int d^4x e^{iqx} \langle J(0) J^\dagger(x) \rangle$$

where the current couples to the state  $|G\rangle$  of the bound state of gluons for our case as

$$\langle 0 | J | G \rangle = F_G M_G^{N-2}.$$

Here  $Q^2 = -q^2$ ,  $N$  is the dimension of the current  $J$ ,  $F_G$  is the decay constant and  $M_G$  is the mass of the state. To construct SR, we follow the work [4] and the study of the glueball SR [5]. The corresponding SR has the following form:

$$\frac{1}{\pi} \int_0^{s_0} \frac{\text{Im}\Pi_{(\text{OPE})}(-s)}{s + Q^2} ds = \frac{F_G^2 M_G^{2(N-2)}}{M_G^2 + Q^2}, \quad (1)$$

where  $\Pi_{(\text{OPE})}(-s)$  is the result of the OPE calculation of the correlator and  $s_0$  is the continuum threshold. In the work [3] dealing with the  $0^{--}$  glueball state in QCD SR, the authors used the following gauge dependent current

$$J_1(x) = g_s^3 d^{abc} \eta_{\alpha\beta} \tilde{G}_{\mu\nu}^a(x) \partial_\alpha \partial_\beta G_{\nu\rho}^b(x) G_{\rho\mu}^c(x), \quad (2)$$

where

$$\eta_{\alpha\beta} \equiv g_{\alpha\beta} - \partial_\alpha \partial_\beta / \partial^2, \quad (3)$$

$G_{\mu\nu}^a$  is the field strength tensor and  $\tilde{G}_{\mu\nu}^a \equiv G_{\alpha\beta}^a i\epsilon_{\mu\nu\alpha\beta}/2$ . They obtained in the Leading Order (LO) part of the correlator as

$$\Pi_{J_1, \text{LO}}^{[3]} = \frac{487}{2^6 3^3 11 \cdot 13 \pi} \alpha_S^3 Q^{12} L, \quad (4)$$

where  $\alpha_S$  is the coupling constant, and  $L = \ln(Q^2/\mu^2)$  and  $\mu^2$  is the momentum scale.

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It is easy to verify that the imaginary part of Eq. (4)  $\text{Im}\Pi_{\text{LO}}$  is negative. Therefore, the right-hand and left-hand sides of Eq. (1) have opposite signs so that this SR becomes inconsistent. Furthermore, we would like to mention that even the numerical factor of logarithm for the LO contribution with the current, Eq. (2), is wrong in [3]. The correct LO result [14] is (log-part only):

$$\Pi_{J_1, \text{LO}}^{\text{here}} = \frac{31}{2^6 3^3 7 \pi} \alpha_s^3 Q^{12} L. \quad (5)$$

However, even this correction does not lead to consistency of SR [15]. We found that the negative sign is related to the structure of the interpolating current, Eq. (2), which induces additional nonphysical poles in the correlator coming from the second term in Eq. (3). Our final conclusion is that the current, Eq. (2), does not couple to any gluonic bound states and should not be considered in the glueball studies.

In this Letter we propose a new gauge invariant current which couples to the  $0^{--}$  three-gluon state. It has the general form:

$$J(x) = \frac{2}{3} g_s^3 \epsilon^{ijk} \text{Tr}((O_i G_{\mu\nu}(x))(O_j G_{\nu\rho}(x))(O_k G_{\rho\mu}(x))), \quad (6)$$

where the operators  $O_m$  are a product of covariant derivatives  $O_i = D_{\alpha_1} \cdots D_{\alpha_n}$ . The lowest dimensional current in this form, that has nonzero LO perturbative contribution to SR corresponds to:

$$\begin{aligned} O_1 G_{\mu\nu}(x) &= D_{\alpha_1} D_{\alpha_2} D_{\alpha_3} \tilde{G}_{\mu\nu}(x), \\ O_2 G_{\mu\nu}(x) &= D_{\alpha_1} D_{\alpha_2} G_{\mu\nu}(x), \\ O_3 G_{\mu\nu}(x) &= D_{\alpha_3} G_{\mu\nu}(x). \end{aligned}$$

The coefficient in the current Eq. (6) was chosen to have the leading term in the following form:

$$J(x) \stackrel{\text{LO}}{=} g_s^3 d^{abc} \tilde{G}_{\mu\nu; \tau_1 \tau_2 \tau_3}^a(x) G_{\nu\rho; \tau_1 \tau_2}^b(x) G_{\rho\mu; \tau_3}^c(x), \quad (7)$$

where  $G_{\mu\nu; \tau_1 \tau_2 \cdots \tau_n}^a = \partial_{\tau_1} \partial_{\tau_2} \cdots \partial_{\tau_n} G_{\mu\nu}^a$ . Using this current, Eq. (6), and calculating the OPE, we have the correlator up to the dimension-8 operators as

$$\begin{aligned} \Pi_{(\text{OPE})}(Q^2) &= \Pi_{(\text{pert})} + \Pi_{(\text{G3})} + \Pi_{(\text{G4})} + \cdots = \\ &\quad \frac{-5\alpha_s^3}{11!4\pi} Q^{20} L \\ &\quad + \frac{-5\pi\alpha_s^3}{3^3 2^5} Q^{14} \left( \langle gG^3 \rangle - \frac{\langle J^2 \rangle}{4} (5 + 2L) \right) \\ &\quad + \frac{205\pi^2 \alpha_s^2}{2^6 3^2} Q^{12} L \langle \alpha_s^2 G^4 \rangle + \cdots, \end{aligned}$$

where the dimension-6 condensates are  $\langle gG^3 \rangle = \langle g f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c \rangle$  and  $\langle J^2 \rangle = \langle J_\mu^a J_\mu^a \rangle$  with the quark current  $J_\mu^a = \bar{q} \gamma_\mu t^a q$ , where the dimension-8 condensate is

$$\langle \alpha_s^2 G^4 \rangle = \langle (\alpha_s f^{abc} G_{\mu\nu}^b G_{\alpha\beta}^c)^2 \rangle - 2 \langle (\alpha_s f^{abc} G_{\mu\nu}^b G_{\nu\beta}^c)^2 \rangle.$$

In contrast to Eq. (5) we now have the positive LO imaginary part and, therefore, the consistency of SR is expected. We would like to emphasize also that so-called direct instantons, which effect strongly SR for the  $0^{++}$  and  $0^{-+}$  two-gluon states [5–7], do not contribute in our case due to the specific color structure of the current, Eq. (6).

After the Borel transformation we have

$$\begin{aligned} \mathcal{R}_0^t(M^2, s_0) &= \frac{1}{\pi} \int_0^{s_0} ds \text{Im}\Pi_t(-s) e^{-s/M^2}, \\ \mathcal{R}_0^{(\text{res})}(M^2, s_0) &= M_G^{20} F_G^2 e^{-M_G^2/M^2}, \end{aligned}$$

where  $M^2$  is the Borel parameter,  $t$  denotes a different contribution to OPE of the correlator: perturbative term (pert), dimension-6 (G3) and dimension-8 (G4) nonperturbative terms. To extract mass from SR, we use a family of derivative SR obtained by differentiation with respect to the Borel parameter  $M^2$ :

$$\mathcal{R}_k^t(M^2, s_0) = M^4 \frac{d}{dM^2} \mathcal{R}_{k-1}^t(M^2, s_0).$$

We define the difference of the OPE result and continuum contribution as

$$\begin{aligned} \mathcal{R}_k^{(\text{SR})}(M^2, s_0) &= \\ \mathcal{R}_k^{(\text{pert})}(M^2, s_0) &+ \mathcal{R}_k^{(\text{G3})}(M^2, s_0) + \mathcal{R}_k^{(\text{G4})}(M^2, s_0). \end{aligned}$$

Then the master sum rules ( $k = 0$ ) and their derivative SRs ( $k > 0$ ) can be expressed by the following equations:

$$\mathcal{R}_k^{(\text{SR})}(M^2, s_0) \approx \mathcal{R}_k^{(\text{res})}(M^2, s_0). \quad (8)$$

The fiducial window  $M^2 \in [M_-^2, M_+^2]$  is limited by the conditions

$$\begin{aligned} |\mathcal{R}_k^{(\text{G4})}(M^2, \infty)| / \mathcal{R}_k^{(\text{SR})}(M^2, \infty) &< 1/3, \\ \frac{|\mathcal{R}_k^{(\text{res})}(M^2, s_0)|}{\mathcal{R}_k^{(\text{SR})}(M^2, \infty)} &\approx \frac{|\mathcal{R}_k^{(\text{SR})}(M^2, s_0)|}{\mathcal{R}_k^{(\text{SR})}(M^2, \infty)} > \frac{1}{10}. \end{aligned} \quad (9)$$

Then the QCD SR for the mass and decay constant can be presented in the form:

$$\begin{aligned} M_G^k(M^2, s_0) &= \sqrt{\frac{\mathcal{R}_{k+1}^{(\text{SR})}(M^2, s_0)}{\mathcal{R}_k^{(\text{SR})}(M^2, s_0)}} \\ F_G^k(M^2, s_0) &= \frac{\sqrt{e^{M_G^2/M^2} \mathcal{R}_k^{(\text{SR})}(M^2, s_0)}}{M_G^{10}} \end{aligned} \quad (10)$$

We define the mass and decay constant by minimization of the criteria  $\delta_k(s_0^{\text{bf}}) = \delta_k^{\text{min}}$  with respect to the threshold  $s_0$  and find the best fit value  $s_0^{\text{bf}}$ :

$$\delta_k(s_0) = \frac{\max |M_G^k(M_i^2, s_0) - M_G^k(s_0)|}{M_G^k(s_0)},$$

$$M_G^k(s_0) \equiv \frac{1}{n+1} \sum_{i=0}^n M_G^k(M_i^2, s_0),$$

where we consider  $n = 20$  points in the fiducial interval  $M_i^2 = M_-^2 + (M_+^2 - M_-^2) i/n$ . In Fig.1, we present the results for the glueball mass and decay constant as a function of the Borel parameter. As one can see, we have a rather good stability plateau for both quantities.

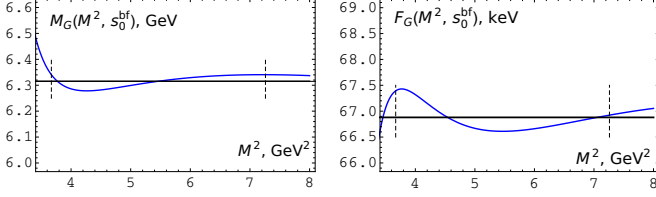


FIG. 1: The Borel parameter dependence of the mass (left panel) and decay constant (right panel) for the central value of the gluon condensate. The vertical lines denote the fiducial interval of Borel parameter.

Finally, we define the decay constant and mass as an average in the fiducial interval for the best fit value of the threshold:

$$M_G = M_G^k(s_0^{\text{bf}}), \quad F_G^2 = \frac{1}{n+1} \sum_{i=1}^n \frac{e^{M_G^2/M_i^2}}{M_G^{20}} \mathcal{R}_k^{(\text{SR})}(M_i^2, s_0^{\text{bf}}).$$

Here we follow the common practice of RG improvement of SR; therefore, in  $\text{Im}\Pi_t(-s)$  all coupling constants are replaced by  $\alpha_s \rightarrow \alpha_s(M^2)$ . We use the strong coupling constant

$$\alpha(Q^2) = \frac{4\pi}{b_0 \ln(Q^2/\Lambda_{\text{QCD}}^2)},$$

with  $b_0 = 11 - 2N_f/3$  and QCD scale  $\Lambda_{\text{QCD}} = 300$  MeV. Working in gluodynamics, we put  $N_f = 0$  and eliminate quark and quark-gluon condensate contributions. The hypothesis of vacuum dominance yields the relation

$$\langle \alpha_s^2 G^4 \rangle = \frac{3}{24} \langle \alpha_s G^2 \rangle^2$$

for the dimension-8 gluon condensate. In this case, the mass of the exotic glueball is determined by the value of the gluon condensate. Unfortunately, this value is not well fixed. According to different analyses carried out in [8–11], we take

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012 \pm 0.006 \text{ GeV}^4.$$

Our final result is

$$M_G = 6.3_{-1.1}^{+0.8} \text{ GeV}, \quad F_G = 67 \pm 6 \text{ keV}. \quad (11)$$

The SR analysis in full QCD ( $N_f = 3$ ) and nonzero quark condensate  $\langle J^2 \rangle$  leads to a reduction of the glueball mass

by 0.2 GeV. The mass of the exotic glueball in Eq.(11) is not far away from the recent unquenched lattice result  $M_G = 5.166 \pm 1.0$  GeV [12] obtained with a rather large pion mass  $m_\pi = 360$  MeV.

Here we would like to note that there are three sources of uncertainties in the above analysis for the mass and decay constant: the variation of gluon condensate, stability of SR triggering Borel parameter  $M^2$  dependence in terms of criteria  $\delta_k^{\text{min}}$ , and roughly estimated SR uncertainty coming from the OPE truncation. The latter uncertainty for the decay constant comes from the definition of the fiducial interval, Eq. (9), in the standard assumption that the contribution from missing terms is of the order of the last included nonperturbative term squared:  $(1/3)^2 \sim 10\%$ . The same error for mass can be expected to be suppressed since the related errors for  $\mathcal{R}_{k+1}^{(\text{SR})}$  and  $\mathcal{R}_k^{(\text{SR})}$  are correlated. The best threshold value is  $s_0^{\text{bf}} = 52.4_{-16.2\%}^{+12.6\%} \text{ GeV}^2$  when only uncertainty of the gluon condensate is included. Note that the fiducial interval for the central value of the gluon condensate is  $M^2 \in [3.7, 7.3] \text{ GeV}^2$ . We also mention that here we present the results from the  $k = 0$  case for SR (see Eqs.(8,10)). The mass estimation for higher values of  $k = 1, 2, 3$  are in agreement with the considered  $k = 0$  case within the error bars.

Summarizing, we present the revision of the QCD SR result for the exotic three-gluon glueball state with quantum numbers  $J^{PC} = 0^{--}$ . A new interpolating current for this glueball has been constructed. By using this current, we have analyzed the QCD SR consisting of contributions up to the operators of dimension-8 and obtained the estimation of the mass and decay constant of the exotic glueball.

After the paper was completed we were informed about the negative result of the searching of the low mass exotic  $0^{--}$  glueball by the Belle Collaboration [13].

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